The Nature of the divergences in Perturbation Theory

Two questions

(1) are renormalized quantities themselves finite and not just a divergent sum of finite terms?

Model calculations ($\lambda\phi^3$ scalar theory) by Heisenberg (1952 & 1953) showed

n^{th} order term $\sim \lambda^n n^{n/2} \cdot n^{-2}$

↑ n^{th} of terms ↑ Power trend on each term

So series diverges.

But n^{th} order term could be conditionally convergent to zero by cancellations of signs.

Probably series is asymptotic.

Typically error involved is smaller than last term calculated, but after a while the last term starts to increase since series actually diverges as $n \to \infty$.

Two arguments for asymptotic nature of Feynman:

(1) Dyson: $e \to ie$ series cannot be expected to converge or indeed to correspond to "a well-defined function" since repulsion of opposite charges could lead to an "explosive disintegration of the vacuum by spontaneous polarization."

(2) Excellent agreement between theory & experiment would indicate previous asymptotic (Heisenberg (1952))

(2) Are the renormalization constants finite?

They may actually be finite and appear infinite because the relevant functions are not analytic.

e.g. $e^{-e^2 \times L} = 1 - e^2 L + \frac{1}{2} e^4 L^2 - \cdots$

as $L \to \infty$, L.H.S. $\to 0$ but each term in the formal perturbation series is infinite.

Haag's theorem shows that Dyson's U-operator, for finite times, used by him to link the Heisenberg & Schrödinger representations does not exist.

Barton (1963) says "Such the fact is never the occurrence of formal divergences is to be expected, and should in no way surprise us".

Ramon (1969) Comments "We may now wonder why, in spite of its non-existence, the interaction picture leads, at least in perturbation theory to reasonable results. The 'mystery' vanishes, in a sense, the understanding manipulations of ordinary Fock quantum mechanics.

Haag's theorem says: sets of spaces appropriate to free fields & interacting fields cannot belong to equivalent representations of the canonical equal-time commutation relations.

The existence of non-equivalent representations is linked to the fact dimensionality of the Hilbert space is non-denumerably infinite as the spaces non-separable. (cf also Schwinger's theory developed in late 1960's.)

Lecture 18

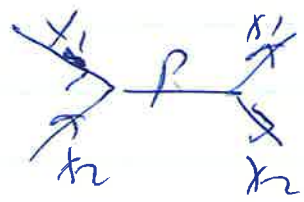
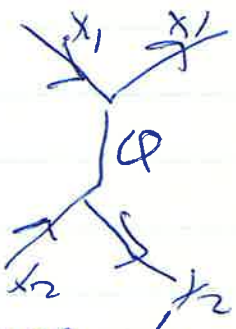
Partial bootstraps

(1) $e = \Pi\Pi(e)$

(2) $N = \Pi N(N)$
$\Pi = NN(\Pi)$

(3) $N = \Pi N(N^*)$ Reciprocal bootstrap
$N^* = \Pi N(N)$

In general $\{P_1 P_2 \ldots\} = X_1 X_2 \ldots \{Q_1 Q_2 \ldots\}$

So  is denoted 

and $\{P\}$, $\{N\}$, $\{\Omega\}$ are all same set.

~~A = B + C~~

Anomalies

$A = B + C$ } A is part of B
$B = A + C$ } and B is part of A.

or $A = A + B$ — A is part of itself.

Resolution of anomalies

Energy in incident system is used to "create" energy particle, not to "release" what is already there.

e.g. $X + A = (B + C) + X$

or not to interpret as A being broken up into B + C by collision with particle X.

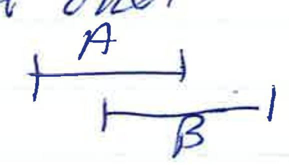
Comparison with other philosophers

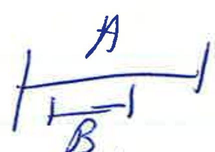
(1) Leibniz   Here stresses analogy with Principle of sufficient reason
"Nature is as it is because it is the only possible world consistent with itself".
i.e. nothing in nature is arbitrary.
But for Leibniz there are many possible worlds allowed by mere consistency. Existence is not itself determined by consistency but is caused by God choosing best possible world
(But cf Russell on Leibniz's secret philosophy)

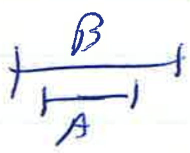
(2) Anaxagoras claims any substance contains any other substance.

But A's view is essentially a containment model.
Anomalies avoided by formulation "seeds contain portions of all seeds".

cf. 2 sets A & B may each include subsets of each other



But if 

then 

is impossible.

Also for Anaxagoras atoms are immutable

Bootstrap is much closer to view that substances can change their forms
cf. Thales, Anaximenes, Anaximander and especially Heraclitus.
Anaxagoras is much closer to Empedocles or Leucippus & Democritus.

(2) Analogies with Eastern Philosophy
such as Hinduism & Buddhism
cf. "The Tao of Physics" by Capra (1975)

(3) Whitehead 's Staff (1971) draws analogy
with a web philosophy of interlocking
process- of Whitehead's Process and
Reality.

Shortcomings of the Bootstrap

1. It makes life very difficult for physicists

2. Partial bootstraps may be impossible
to satisfy — we may have to include
all particles at once

3. Idea may be untestable, and so unscientific
because of computation gap

4. Bootstrap does not include elementary
of photon — then claims other particles
are associated with the process of
measurement, to should be treated
differently.
But what about the other leptons
— the muon & the neutrino.

5. Then what are valid metaphysical
claims. We say complete bootstrap
would demand for self-consistency
" confronting the classic concept of "
measurement and possibly even of consciousness
(1968, Second)

Tuesday

(19)

The Ultimate Nature of Matter

1. Bootstrap Models

$$X_1 = \begin{cases} X_1 \\ X_a + X_2 \\ \vdots \end{cases}$$

$$X_2 = \begin{cases} X_2 \\ X_a + X_2 \\ \vdots \end{cases}$$

2. Thalean Fundamentalism

$$X_1 = Q_1 + Q_2 + \cdots$$
$$X_2 = Q_1 + Q_2 + \cdots$$
$$\vdots$$

or simple constituent model of quarks or partons

3. Anaximandrian Fundamentalism

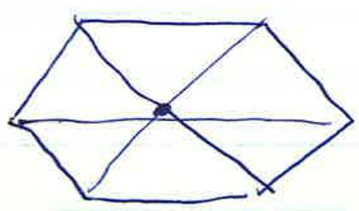
$$X_1 = X_1(F)$$
$$X_2 = X_2(F)$$
$$\vdots$$

cf Heisenberg's unified field theory

cf knots on a string (Rattlers 1971)

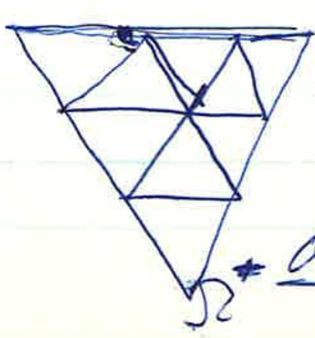
4. Mathematical Atomism

Analogy between Plato's theory in Timaeus of building regular solids from 2 sorts of triangle and the SU(3) symmetry patterns being built up out of simple triangular representations.



octet

or



N* resonances

Ω^* decuplet

Conclusions

1. Analyses of correspondence relations

Emphasizes the speculative element in theories. Our analysis has been in terms of a directional shift followed by a "stretching" in the direction indicated by a polarizing phenomenon or by a polarizing property of some model of the old theory.
(This elaborates on the procedure followed in my Synthese paper.)

2. Role of surplus structure

(a) ~~The way of a~~ Reformulation involves changing the surplus structure in which the theory is embedded.

(b) "Stretching" may involve ~~of~~ introducing a new axiom which operates in the surplus structure (cf analyticity postulate)

(c) In some cases surplus structure comes to be afforded ontological reference (cf Fock space), but in other cases this does not happen (cf Analytic S-matrix theory).

(d) Emphasis on purely mathematical considerations is a feature of modern theoretical

Physics — of Einstein & Dirac

(Both have been stressed by Zahar using as examples. Maxwell's introduction of displacement current
Einstein's use of Riemannian geometry
Weyl's interpretation of his transformation eqns in terms of a gauge centroclinic
Dirac's interpretation of -2 energy solns as positrons etc.)

Quotation from Dirac (1931)

"Advance in physics is to be associated with a continual modification & generalization of the axioms at the base of the mathematics rather than with a logical development of any one mathematical scheme on a fixed foundation" [or was reflected in the last century]
ex- Non-Euclidean geometry & relativity
non-commutative algebra in QM

3. The Problem of the Computation Gap — The Floating Model

If an approximate calculation disagrees with experiment we do not know whether to correct the modifications at the original theory or at the approximation
In the case of atomic & molecular physics we have some confidence in the

in the underlying theory because there are simple calculable problems, such as the hydrogen or helium atom, or the hydrogen molecule, which can be solved very accurately so these predictions really do test the theory and not the theory + approximation.

We can now argue that it is a more complicated problem (in nuclear physics or chemistry for example) a choice of approximation gives results in agreement with experiment then we may believe this approximation has picked out the essential relevant features of some complicated dynamical situation & we can justify our approximation a posteriori in virtue of its success.

But in particle physics of strong interactions there are no simple calculable problems — this is the message of the bootstrap.

Hence our approximation models are not picked out by any plausible underlying theory — in this sense they may be said to float.

(Contrast Post's sense of floating models in which approximate models do not agree with observation — they float at both ends (theoretical & observational) But such models are only objectionable if they are not allied with a refinement that the mismatch with experiment is describable (theoretically)

in a simple way - cf Elliot's SU(3) in nuclear physics
where intermediate subgroups to symmetries is assumed to be a function of the dynamics of the symmetry. So there is no meaning of different irreducible representations.

4. The failure of Scientific Method

The bootstrap philosophy if correct, would point to a fundamental impasse for scientific method as we know it.
The success of scientific methods depends on the possibility of being able to isolate phenomena and of being able to disregard the enormous complexity of real-life situations.
The bootstrap philosophy would tell us that in the realm of hadron dynamics this approach is no longer possible. As a parallel one can think that celestial mechanics would have been like if the planetary system was not susceptible to perturbation theory.
The bootstrap philosophy is essentially one of despair and frustration although chew herself sees things just the other way around (1970)

" I would find it a cruel disillusionment if in 1980 all of hadron physics could be explained in terms of a few arbitrary entities - We should then be in essentially the same posture as in 1930 ... to have learned so little in half a century would be the ultimate frustration "

TABLE of the ELEMENTARY PARTICLES

BOSONS	FERMIONS
150 π (pion) 500 K (kaon) 550 η + resonances 750 ρ 800 ω ⋮ 3,100 } 24 3,700 }	HADRONS MESONS BARYONS nucleons { proton, neutron 950 hyperons { Λ 1100, Σ 1200, Ξ 1300, Π 1650 + resonances Δ(1236) ⋮
○ photon ○ graviton (?) ~~2000~~ 10,000 W-particle (?)	LEPTONS { ν_e (neutrino) ○, ν_μ (neutretto) ○, e (electron) ½, μ (muon) 100
Quarks (?) ~~5000~~ 25,000	

(Rest energies in Mev rounded to nearest 50 Mev)

about 500 particles in all.

The Analytic S-Matrix

Two threads in research programme of the analytic S-Matrix.

(1) What should fundamental theory of elementary particles refer? — to S-Matrix

(2) a new non-perturbative method of calculating the S-Matrix.

Heisenberg (1943) introduced S Matrix (cf Wheeler (1937)) by asking 2 questions

(1) If a "complete" theory includes a fundamental length what features of existing theories would survive (cf Einstein approach to Special Relativity)

(2) Should not a theory anyway restrict itself to what can actually be observed (cf. HS approach to Matrix Mechanics)

Heisenberg answered both questions with his S-Matrix which provides 2 sets of information

(1) Scattering & reaction cross-sections

(2) bound states and resonances (short lived unstable particle states) related to singularities in the S-Matrix

Non-Perturbative Calculations of the S-Matrix

Success of Q.E.D. due to small value $e^2/\hbar c$
(measures mean no virtual particles in "cloud")
this allows perturbation theory.

For hadron physics (the vast majority of ~500 particles) we are dealing with strong interactions so perturbation theory does not apply.

Four Basic Interactions

Transparency (12)

strong ← cf atomic bomb
Electromagnetic ← chemical high explosive
weak
gravitational

Transparency (13)

(Table of the Particles)

Approximation schemes

1. Tamm Dancoff: limited no particles in arbitrary no of states
2. Tomonaga: arbitrary no of particles in a limited no of states.

Both these deal with virtual particles
Much more successful approach to low energy πN scattering achieved by

3. Chew-Low-Wick: Meson scattering

amplitudes for a real (renormalized) physical process in terms of scattering amplitudes for all real processes which could connected with both the initial & final states

Oehme (1955) showed that Chew-Low model was an example of a dispersion relation and was connected with analyticity properties of the S-Matrix.

Dispersion Relations

Dispersion relations approach to calculating the S-matrix involves the following sequence of ideas.

(1) Consider S-matrix elements as functions of energy (& other variables such as momentum transfer) and now allow energy to assume complex values.

(2) Assume S-matrix is an analytic function except for certain singularities (By Cauchy's theorem a bounded analytic function with no singularities is an integer) a constant.)

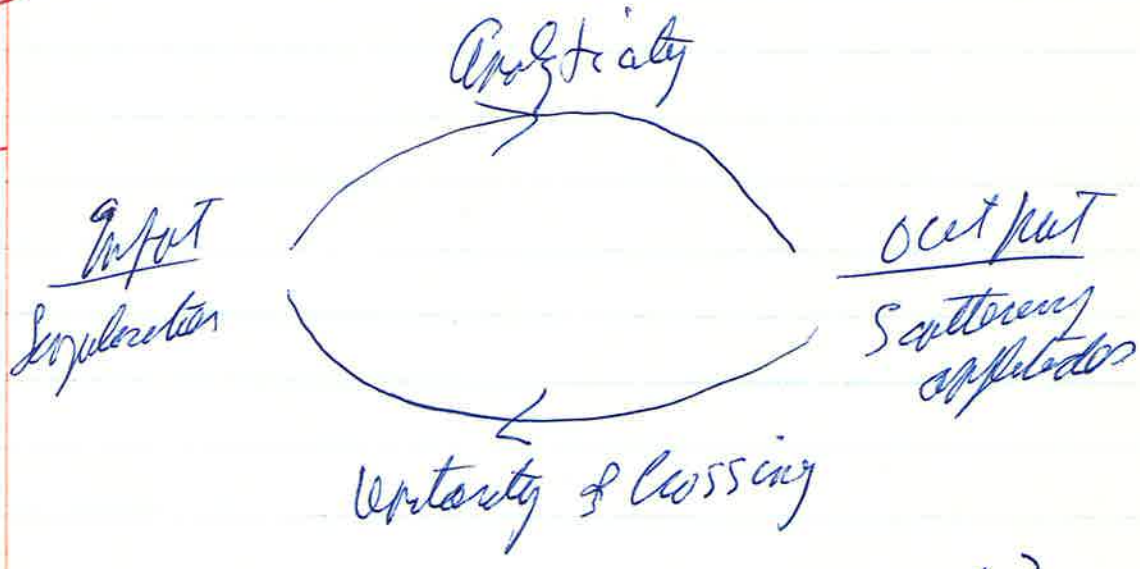
(3) Use Cauchy's Theorem to relate scattering amplitudes to the singularity structure (poles of singularities & behaviour in their neighbourhood - residues at poles, discontinuities across branch cuts).

(4) We use unitarity & crossing principles to locate part of the singularity structure

(5) We assume there are no other singularities than those demanded by unitarity & crossing — Principle of Maximal analyticity of the first kind or the Mandelstam conjecture.

(6) We now have feedback relation.

Transparency (14)



$$\text{or} \quad \left.\begin{aligned} Y &= X \\ X &= Y^2 \end{aligned}\right\} \quad Y = 0 \text{ or } 1$$

(7) We look for possible ambiguities in solving integral equations and seek to remove them by a principle which depends certain singularities. This leads to the Chew-Frautschi Principle of Maximal analyticity of the 2nd kind

then is defined in term of analyticity in Complex angular momentum plane. 10.
every particle pole is a Regge pole 10. lies on a Regge trajectory in the Complex angular momentum plane.

(In our model if poln is to be symmetric under reflection in the origin $\Rightarrow \nu = 0$)

This principle leads to the ~~Chew-Frautschi~~

Chew-Frautschi Bootstrap Hypothesis (1961)

there are no "arbitrary" particle poles which all poles exist to identified with the elementary particles (aristocrats)

all poles are determined by self-consistency and are dynamical in origin in view of the fact that the location of the poles depends on the strength of the interaction. (nuclear democracy)

(Note: The principle of Maximal Analyticity of the S-matrix leads to asymptotic absence of C.D.D. poles in Chew-Low-Model. The source of the ambiguity is that the Mandelstam amplitude blows up rather than polynomially at ∞, i.e. asymptotic behaviour. ~~C.D.~~ Elimination of C.D.D. ambiguity is equivalent to assuming vanishing of phase shifts at high energy)

~~Note~~

Principle of Maximum Strength: the field remains the quarter of whether all arbitrary parameters could be eliminated from the theory.
In 1962 Chew & Frautschi introduced a Principle of Maximum Strength to fix the overall strength of the interactions.

(this may not be required as a separate assumption)

The final objective of the bootstrap is to have no arbitrary parameters except a dimensional constant to fix the scale of the hadron masses.

There are 3 possibilities:
(1) several sets of particles that satisfy the bootstrap
(2) No set of particles that satisfy the bootstrap
(3) There is a unique set of particles that satisfy the bootstrap and those are the particles observed in Nature.

The last possibility we may call the Chew-Frautschi Hypothesis.

Transparency (15)

Singularities from Unitarity & Crossing

Reaction amplitudes → Im Scattering amplitude

→ singularity structure of particle poles & normal threshold branch points.

Crossing states that one scattering amplitude may be related to another by analytic continuation to unphysical values of the energy & momentum transfer (in the simplest case of 2-body elastic scattering). Unitarity now forces singularities in these crossed channels which appear as singularities in the direct channel at unphysical values of the energy and momentum transfer arguments

Analyticity & Causality

Transparency (16)

We can derive some analytic properties from causality in classical physics – cf the Kramers-Kronig dispersion relations in optics.

To pursue ideas in RQFT we can try to derive analyticity from micro causality as expressed in the canonical commutation relations

$$[\varphi_1(x_1,t_1), \varphi_2(x_2,t_2)] = 0 \quad \text{if } (x_1,t_1)\to(x_2,t_2) \text{ is space-like interval}$$

But we could rigorously derive analyticity in sufficiently large domains to exploit full power of dispersion relations approach.

We want complete information about analytic properties in all variables

The Status of the Mandelstam Conjecture

Mandelstam (1958) introduced his conjecture by two ~~arguments~~ implicit arguments.

(1) Let us be guided by discovery of such dispersion relations as can be derived from field theory (e.g. forward scattering of π-N system)

(2) Let us use what we can know from examination of analytic properties of Feynman graphs (Mandelstam had studied some 4th order Feynman graphs in detail to this purpose)

In his 1958 paper Mandelstam went beyond his conjecture to assume that the singularities dictated by unitarity would be such as to allow a Sperko representation (in double terms of a double dispersion relation) — the Mandelstam representation

But: Mandelstam himself found that the Sperko representation was not general and in perturbation theory (1959) not even valid for any order field theory for which perturbation theory existed (1960).

However Mandelstam representation is true for non-relativistic potential scattering (by Yukawa, Wells e.g.) [Blankenbecler, Goldberger, Khuri, & Treiman (1960) & Regge (1959)]

Mandelstam himself regarded his conjecture as just that a conjecture as to how field theories should behave. He did not subscribe to axiomatic S-matrix theory because (1) S-matrix contains less than can be potentially measured

(2) analyticity assumptions as axioms "are rather artificial" (1962 lecture)

But Chew & others (Frautschi, Stapp, Landau) took up enthusiastically the idea that dispersion relations could replace field theory (originally proposed by Gell-Mann at Rochester Conference in 1956)

Mandelstam conjecture would play the role of a postulate & axiom (cp Heisenberg 1946)

Chew regards the maximal analyticity postulate as the mathematical expression of a fundamental principle of maximum smoothness

In his 1966 book The Analytic S-Matrix Chew emphasizes also the mathematical aspect of the postulate

" In a deep sense physics is based on analytic functions. It is pointless to seek a logical origin for this circumstance. Physical theory cannot be based on logic; it is always a matter of guesswork based on observation of Nature. One cannot, for example, argue that it is

logical for classical mechanics to be expressible through second order differential equations. This really is the scheme that works!"
(cf Fourier who had enabled physics to deal with non-analytic functions!)

So Chew stresses an axiom which is concerned essentially with the "simpler structure"

(note Stapp has discussed links between macro causality and normal analytic structure in the physical region. But maximal analyticity postulate is still required to extend the primitive domain of analyticity so cased by the macro causality Principle).

Transparency The Chew-Mandelstam heuristic Strategem

17

(1) Mandelstam derives a property of analyticity by considering an approximation, models of field theory (in 4th order perturbation theory)

(2) Mandelstam conjectures this property may be true of complete theory

(3) Chew now reinterprets the conjecture as being "model-independent" by fiat and takes it as an axiom for

a new theory which may or may not be equivalent to the old theory.

(17) Introduce them theoretically in two ways.

T is RQFT
A is Feynman perturbation expansion
T_1 is a class of Feynman diagrams
P is analytic property of these diagrams
C is dispersion relations connecting observable quantities which are verified experimentally
T'(P) is a theory S-matrix theory

(Another example of this theory is current algebra (Gell-Mann (1962)))